Journal of Computer and Systems Sciences International, Vol. 39, No. 2, 2000, pp. 237–243.
Translated from Izvestiya Akademii Nauk. Teoriya i Sistemy Upravleniya, No. 2, 2000, pp. 78–84.

Guaranteed Optimization of Uncertain Systems Through the Use of Prediction

N. V. Balashevich, R. Gabasov, and F. M. Kirillova

Institute of Mathematics, Belarussian Academy of Sciences, Belarussian State University, ul. Leningradskaya 14, Minsk, 220080 Belarus

Received November 3, 1994; in final form, August 31, 1998

Abstract—A new type of special optimal state feedback is introduced for optimizing dynamical systems under conditions of constantly acting disturbances. Using a priori and current information, possible disturbances are represented as the sum of sufficiently regular and irregular components. An algorithm of operation of a controller is described; this controller processes the trajectory executed and calculates the values of an optimal feedback control for each particular control process in a real-time mode. The results are illustrated by an example.

0. INTRODUCTION

The construction of feedbacks for dynamical systems with the required properties of the transients in these systems remains to be a central problem in control theory. This problem has received the largest developmental effort in the conventional linear stabilization theory [1]. In modern stabilization theory, linear feedbacks were supplemented by bang-bang feedbacks with linear switching surfaces; this has resulted in the creation of stable systems with variable structures [2, 3]. An interesting trend in the feedback theory has been evolving recently within the framework of the H_∞-control theory [4]. Here, a linear feedback is chosen in such a way that a linear dynamical system closed by this feedback is asymptotically stable and minimizes a special norm of the operator that is realized by the closed-loop system when disturbances pass through it.

A new stage in the development of feedback theory has come with the emergence of optimal control theory [5, 6]. At this stage, it became possible not to assign the structure of the feedback in advance, but to find it in general from an optimality condition for a chosen performance index. The first examples of synthesis of time-optimal feedbacks for stationary two-dimensional systems raised hopes that this problem could be solved in the general case. However, for multidimensional systems, a certain success has been achieved only in the case of linear-quadratic control problems introduced by A.M. Letov [7] and R.E. Kalman [8]. The results of A.M. Letov and R.E. Kalman have found extensive application and have served as a source of numerous studies dealing with the problem of optimal feedbacks. Not only deterministic models were studied within the framework of this trend of optimal control theory, but systems with uncertainty as well. Naturally, the problems of optimal observation (filtration) were solved along with the optimal control problems in this process. Features peculiar to the class of problems under consideration required rather severe constraints to be imposed on the elements of problems under study. The most prominent among them were the absence of direct constraints on the control and the special nature of disturbances that were described by Gaussian stochastic processes as a rule.

The optimal feedbacks that were introduced and constructed for the first time-optimal control problems can be called conventional. Their main feature consisted in the fact that they were synthesized for deterministic models, although these feedbacks were designed for the functioning of closed dynamical systems under conditions of constantly acting disturbances. Such a type of feedback was also common in the conventional control theory. It was already mentioned that the optimal feedbacks for the mathematical models with disturbances were originally constructed within the framework of theory of linear-quadratic control problems. In addition, the disturbances considered were assumed to be stochastic [9]. The optimal feedback theory for systems with nonstochastic disturbances began to evolve after the creation of differential games theory [10]. This approach naturally led to the fact that this new optimal feedback theory was aimed at obtaining guaranteed results rather than at obtaining the results that were optimal in the mean (in the probabilistic sense) [11].

As is known, the construction of optimal feedback is eventually reduced to solving partial differential equations that generalize conventional Bellman equations for deterministic optimal control problems. In spite of the introduction of special approximation schemes and the development of the theory of generalized Bellman equations, the actual construction of guaranteeing optimal feedbacks is still fraught with great difficulties; the most important among them is the dimensionality problem.

The analysis of the use of conventional feedback in real control processes shows that, in each process, the values of an optimal feedback control are needed only along an isolated trajectory. These values are needed not at once, but in the course of a control process and at a pace at which real time goes on. Taking into account the finiteness of the rate at which the real time goes on and the internal performance of modern computers, it is possible to establish a correspondence between the rate of progress of processes being optimized and the internal performance of computing devices that is necessary for the optimization of these processes.

Consider the following simplest optimal control problem:

$$J(u) = c'x(t^*) \longrightarrow \max, \quad \dot{x} = Ax + bu,$$
$$x(0) = x_0, \quad Hx(t^*) = g, \quad |u(t)| \le 1, \tag{0.1}$$
$$t \in T = [0, t^*], \quad x \in R^n, \quad g \in R^m,$$
$$\operatorname{rank} H = m < n,$$

where $u(t)$ is a scalar control, A, H, b, c, and g are given matrices and vectors of the corresponding dimensions, t^* is a fixed instant. Although the elements of the problem are linear, it contains all the principal elements of the general optimal control problem. The linear performance index $J(u)$ is sufficiently general; an active use of this performance index in optimal control theory dates back to papers by L.I. Rozonoer [12]. The solution of the synthesis problem for (0.1) can be obtained as follows. We imbed problem (0.1) into a family of problems of the form

$$J(u) = c'x(t^*) \longrightarrow \max, \quad \dot{x} = Ax + bu,$$
$$x(\tau) = z, \quad Hx(t^*) = g, \quad |u(t)| \le 1, \tag{0.2}$$
$$t \in T^\tau = [\tau, t^*).$$

Let $u^0(t|\tau, z)$, $t \in T^\tau$ be an optimal open-loop control in problem (0.2) for a fixed pair (τ, z), and let X_τ be the set of all vectors $z \in R^n$ for which problem (0.2) has a solution with fixed τ. The conventional feedback is defined by the following relation:

$$u^0(\tau, z) = u^0(\tau|\tau, z), \quad z \in X_\tau, \quad \tau \in T. \tag{0.3}$$

We close system (0.1) and consider this system in the presence of the unknown disturbances of the form

$$\dot{x} = Ax + bu^0(t, x) + w(t), \quad x(0) = x_0.$$

It is clear that in a particular process $x^*(t)$, $t \ge 0$ (under a certain realized disturbance $w^*(t)$, $t \ge 0$), the value

$$u^*(t) = u^0(t, x^*(t)), \quad t \in T,$$

is input into the system, i.e., we do not need the whole optimal feedback (0.3) in the real process, but only the values of this feedback along the forming curve $x^*(t)$, $t \in T$. The value $u^*(\tau) = u^0(\tau, x^*(\tau))$ need not be known in advance; it is sufficient to calculate this value at the instant τ when the system is at the point $x^*(\tau)$. It turns out that the procedure for calculating $u^*(t)$, $t \ge 0$, can be arranged in a real-time mode [13]. This idea provides the basis of the method for implementing the optimal feedback both for system (0.1), as well as for other problems that were considered by the authors. The same idea provides a background for the approach to solving the guaranteed optimization problem presented in what follows.

One of the guaranteed optimization problems with additive disturbances was considered in [14]. If disturbances acting on a system contain sufficiently regular components, then, processing the formed trajectory, it is possible to approximate these disturbances in the finite-parameter class of functions and use the result as a prediction [15].

The present paper combines the ideas of both approaches [14, 15]. It is assumed that disturbances acting on a system can be represented as the sum of a sufficiently regular part and an irregular part with the known range. In what follows, regular and irregular parts mean the slowly and rapidly changing components of the disturbance, respectively.

1. STATEMENT OF THE PROBLEM

In the class of piecewise continuous controls $u(t)$, $t \in T = [0, t^*]$, we consider the problem

$$J(u) = c'x(t^*) \longrightarrow \max, \tag{1.1}$$

$$\dot{x} = Ax + bu + qw, \quad x(0) = x_0, \tag{1.2}$$

$$h_i'x(t^*) \ge g_i, \quad i \in I = \{1, \dots, m\}, \tag{1.3}$$

$$|u(t)| \le 1, \quad t \in T, \quad (x \in R^n, \ u \in R, \ w \in R), \tag{1.4}$$

where $w(t)$, $t \in T$, is an unknown piecewise continuous disturbance function. Suppose that, based on a priori data and on the analysis of the formed trajectory $x(t)$, $t \in T$, at any current instant $\tau \in T$, the following determinate functions are known: $\tilde{w}_\tau(t) = \tilde{w}(t|\tau, x_\tau(\cdot))$, $t \in T^\tau = [\tau, t^*)$, and also that

$$\tilde{v}_\tau(t) = \tilde{v}(t|\tau, x_\tau(\cdot)), \quad t \in T^\tau,$$
$$\text{and} \quad |\tilde{w}_\tau(t) - w(t)| \le \tilde{v}_\tau(t), \quad t \in T^\tau, \quad \tau \in T. \tag{1.5}$$

We call the function $\tilde{w}_\tau(\cdot) = (\tilde{w}(t), t \in T^\tau)$ the trial signal on the interval T^τ. The methods for constructing the functions $\tilde{w}_\tau(\cdot)$, $\tilde{v}_\tau(\cdot)$, $\tau \in T$, depend on a particular application domain and on the available information. These methods are not considered in the present paper. We only note that the better the trial signal approximates the actual disturbance, the greater is the effect of using this signal, and such signals can be constructed, as a rule, for sufficiently regular disturbances only. The error function $\tilde{v}_\tau(\cdot)$, $\tau \in T$ is introduced in order to take into account the irregular components of disturbances.

Assume that the efficiency of processing the incoming information grows in the course of the control process; i.e., if $\tau_2 > \tau_1$, then, for trial signals $\tilde{w}_{\tau_1}(\cdot)$ and $\tilde{w}_{\tau_2}(\cdot)$, and for any disturbances $\tilde{v}_{\tau_2}(\cdot)$, $|\tilde{v}_{\tau_2}(t)| \le \tilde{v}_{\tau_2}(t)$, $t \in T^{\tau_2}$, there exist the disturbance $\tilde{v}_{\tau_1}(\cdot)$, $|\tilde{v}_{\tau_1}(t)| \le \tilde{v}_{\tau_1}(t)$, $t \in T^{\tau_1}$ such that $\tilde{w}_{\tau_2}(t) + \tilde{v}_{\tau_2}(t) \equiv \tilde{w}_{\tau_1}(t) + \tilde{v}_{\tau_1}(t)$, $t \in T^{\tau_2}$.

A piecewise continuous function $u(t) = u(t|x_0, \tilde{w}_0(\cdot))$, $t \in T$, satisfying inequality (1.4), is called a guaranteeing open-loop control for the trial signal $\tilde{w}_0(\cdot)$ if all the trajectories of system (1.2) generated by this control and by the initial state x_0 reach the terminal set (1.3) for all possible disturbances $w(t)$, $t \in T$, such that $|w(t) - \tilde{w}_0(t)| \le \tilde{v}_0(t)$, $t \in T$.

Set us introduce the notation $v_\tau(t) = w(t) - \tilde{w}_\tau(t)$, $t \in T^\tau$. The number $J_0(u) = \min\limits_{v_0} c'x(t^*)$ is called the value of the performance index (1.1) on the guaranteeing open-loop control $u(\cdot) = (u(t), t \in T)$.

The optimal guaranteeing open-loop control for the trial signal $\tilde{w}_0(\cdot)$ is the guaranteeing open-loop control $u^0(\cdot)$, on which the performance index $J_0(u)$ reaches its maximum, i.e., $J_0^0 = J_0(u^0) = \max\limits_u \min\limits_{v_0} c'x(t^*)$.

The optimal guaranteeing open-loop control in the actual process do not allow one to use the information about the realized states of the system and the trial signals that are adjusted on the basis of these states. That is why, the preference in practice is given to optimal feedback controls.

To introduce the optimal feedback control, we imbed the problem (1.1)–(1.4) into the family of problems

$$J(u) = c'x(t^*) \longrightarrow \max, \quad \dot{x} = Ax + bu + q(\mu + \nu),$$

$$x(\tau) = z,\ h_i'x(t^*) \ge g_i,\ i \in I,\ |u(t)| \le 1,\ |\nu(t)| \le \tilde{\nu}(t),$$

$$t \in T^\tau,$$

which depends on the scalar τ, n-vector z, and on the functions $\mu(t)$ and $\tilde{\nu}(t)$, $t \in T^\tau$.

We denote by Ω the set of all positions $s = (\tau, z, \mu(\cdot), \tilde{\nu}(\cdot))$ for which there exists an optimal open-loop guaranteeing control $u^0(t|s)$, $t \in T^\tau$, $s \in \Omega$, for the trial signal $\mu(\cdot) = (\mu(t), t \in T^\tau)$.

The function

$$u^0(s) = u^0(\tau|s), \quad s \in \Omega, \tag{1.6}$$

is called the optimal guaranteeing feedback control.

Consider the behavior of the dynamical system (1.2) that is closed by the optimal feedback (1.6), i.e.,

$$\dot{x} = Ax + bu^0(t, x, \tilde{w}_t(\cdot), \tilde{v}_t(\cdot))$$
$$+ q(\tilde{w}_t(t) + v_t(t)), \quad x(0) = x_0, \tag{1.7}$$
$$|v_t(t)| \le \tilde{v}_t(t), \quad t \in T.$$

We denote by $w^*(t)$, $t \in T$, the unknown disturbance that has been realized in a certain process of functioning of system (1.7). This disturbance generates the trajectory $x^*(t)$, $t \in T$, of system (1.7). Therefore, we use in this process only the values of the control $u^*(\tau) = u^0(\tau, x^*(\tau), \tilde{w}_\tau^*(\cdot), \tilde{v}_\tau^*(\cdot))$, along the generated trajectory $x^*(\tau)$, $\tau \in T$ of the constructed trial signal $\tilde{w}_\tau^*(\cdot)$, $\tau \in T$ and of the guaranteed error $\tilde{v}_\tau^*(\cdot)$, $\tau \in T$.

The calculation of the function $u^*(t)$, $t \in T$, in the real-time mode is called the implementation of the optimal feedback (1.6) in a particular process, and the device that is capable of performing this calculation is called the optimal controller. Thus, to implement the feedback control (1.6) in each particular control process, it is sufficient to construct the algorithm for operation of an optimal controller.

2. DEFINING EQUATIONS OF AN OPTIMAL CONTROLLER.

By the Cauchy formula, we have

$$x(t^*) = F(t^* - \tau)x^*(\tau)$$
$$+ \int_\tau^{t^*} F(t^* - t)(bu(t) + qw^*(t))dt,$$

where $\dot{F} = AF$, $F(0) = E$. Taking (1.5) into account, we introduce the notation

$$\hat{g}_i(\tau) = g_i - h_i'F(t^* - \tau)x^*(\tau)$$
$$+ \int_\tau^{t^*} |h_i'F(t^* - t)q\tilde{v}_\tau^*(t)|dt - \int_\tau^{t^*} h_i'F(t^* - t)q\tilde{w}_\tau^*(t)dt.$$

By definition, to produce the current value of the control $u^*(\tau)$ at each instant $\tau \in T$, the controller uses the solution to the following problem:

$$J_\tau = \int_\tau^{t^*} c'F(t^* - t)bu(t)dt \longrightarrow \max,$$

$$\int_\tau^{t^*} h_i'F(t^* - t)bu(t)dt \ge \hat{g}_i(\tau), \quad i \in I, \tag{2.1}$$

$$|u(t)| \le 1, \quad t \in T^\tau.$$

Let $u^0_\tau(t)$ and $x^0_\tau(t)$, $t \in T^\tau$ be the optimal open-loop control and the corresponding trajectory of problem (2.1), respectively. We denote by $I^*_\tau = \{i \in I$: $\int_\tau^{t^*} h_i' F(t^* - t)bu(t)dt = \hat{g}_i(\tau)\}$ the set of indices of active constraints of problem (2.1). By the maximum principle (see [6]), the optimal control $u^0_\tau(t)$, $t \in T^\tau$, has the form $u^0_\tau(t) = \max \psi'_\tau(t)b$, $t \in T^\tau$, where $\psi_\tau(t)$, $t \in T^\tau$, is the solution to the adjoint system $\dot\psi = -A'\psi$, $\psi(t^*) = c - \sum_{i \in I^*_\tau} h_i y_i(\tau)$; $y_i(\tau)$, and $i \in I^*_\tau$ are the optimal Lagrange multipliers of problem (2.1). We introduce the function $\Delta^0_\tau(t) = \psi'_\tau(t)b$, $t \in T^\tau$, which is the optimal cocontrol of problem (2.1). Then, the optimal control $u^0_\tau(t)$, $t \in T^\tau$, can be written in the form

$$u^0_\tau(t) = \operatorname{sgn}\Delta^0_\tau(t), \quad t \in T^\tau, \tag{2.2}$$

i.e., the switching points of the optimal control $u^0_\tau(t)$, $t \in T^\tau$ coincide with the zeros of the function $\Delta^0_\tau(t)$, $t \in T^\tau$.

Therefore, if the condition $\operatorname{rank}\{h_i' F(t^* - t)b,\ t \in T^\tau_*,\ i \in I^*_\tau\} = |I^*_\tau|$, where $T^\tau_* = \{t \in T^\tau\colon u^0_\tau(t-0) \neq u^0_\tau(t+0)\}$, holds, then the solution to problem (2.1) is uniquely determined by the values of the elements

$$t_i(\tau), \quad i = \overline{1, p(\tau)}; \quad y_i(\tau), \quad i \in I^*_\tau, \tag{2.3}$$

where $t_i(\tau)$, $i = \overline{1, p(\tau)}$, are zeros of the optimal cocontrol, i.e., $\Delta^0_\tau(t_i(\tau)) = 0$, $i = \overline{1, p(\tau)}$.

The following terminal constraints hold on the optimal control (2.2):

$$\int_\tau^{t^*} h_i' F(t^* - t)bu^0_\tau(t)dt = \hat{g}_i(\tau), \quad i \in I^*_\tau,$$

$$\int_\tau^{t^*} h_i' F(t^* - t)bu^0_\tau(t)dt > \hat{g}_i(\tau), \quad i \in I \backslash I^*_\tau;$$

the corresponding cocontrol $\Delta^0_\tau(t)$, $t \in T^\tau$ vanishes at the points of switching of the optimal control (2.2), i.e., $\Delta^0_\tau(t_i(\tau)) = 0$, $i = \overline{1, p(\tau)}$.

From this, we obtain the following set of equations for the elements defined by (2.3):

$$\begin{gathered}\sum_{i=0}^{p(\tau)} k_i \int_{t_i(\tau)}^{t_{i+1}(\tau)} h_l' F(t^* - t)bdt - \hat{g}_l(\tau) = 0, \quad l \in I^*_\tau, \\ \left(c' - \sum_{i \in I^*_\tau} y_i(\tau)h_i\right)F(t^* - t_j)b, \quad j = \overline{1, p(\tau)},\end{gathered} \tag{2.4}$$

where $t_0(\tau) = \tau$, $t_{p+1}(\tau) = t^*$, $k_i = \operatorname{sgn}\Delta^0_\tau(t_i(\tau) + 0)$.

Equations (2.4) will be called the defining equations of the optimal controller. System (2.4) contains $|I^*_\tau| + p(\tau)$ nonlinear equations in $|I^*_\tau| + p(\tau)$ variables defined by (2.3). Under sufficiently general conditions, the Jacobian matrix of system (2.4) is nondegenerate.

At the instant τ, the structure of equations (2.4) is completely determined by the number $p(\tau)$ of zeros $t_i(\tau)$, $i \in P$, of the cocontrol $\Delta^0_\tau(t)$, $t \in T^\tau$, by the values k_i, $i \in \{0, \ldots, p(\tau)\}$, of the control on the intervals of constancy, and by the set I^*_τ.

The structure of equations (2.4) can change at the instant $\tau = \bar\tau$ due to one of the following reasons:

(1) $t_1(\tau) \longrightarrow \bar\tau$ as $\tau \longrightarrow \bar\tau - 0$;

(2) $t_p(\tau) \longrightarrow t^*$ as $\tau \longrightarrow \bar\tau - 0$;

(3) $p(\bar\tau) < p(\bar\tau - 0)$;

(4) $p(\bar\tau) > p(\bar\tau - 0)$;

(5) for certain $i \in I^*_{\bar\tau - 0}$ $y_i(\bar\tau) = 0$;

(6) $|I^*_\tau| > I^*_{\bar\tau - 0}$.

The following two procedures for solving system (2.4) are used: (1) the procedure for solving system (2.4) on the intervals with a constant structure; (2) the procedure of passage between neighboring intervals with constant structure. The details of these procedures are similar to those presented in [13].

On the basis of the solution of the defining equations at the instant τ, the controller inputs the signal $u^*(\tau) = k_0$ into the system (1.7) if the instant τ belongs to the regular interval of control. The rules of functioning of the optimal controller on the intervals of sliding are described in [13].

3. EXAMPLE

As an illustration, we consider the following control problem for oscillatory motion:

$$\int_0^{2\pi} u(t)dt \longrightarrow \min, \quad \dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + u + w,$$

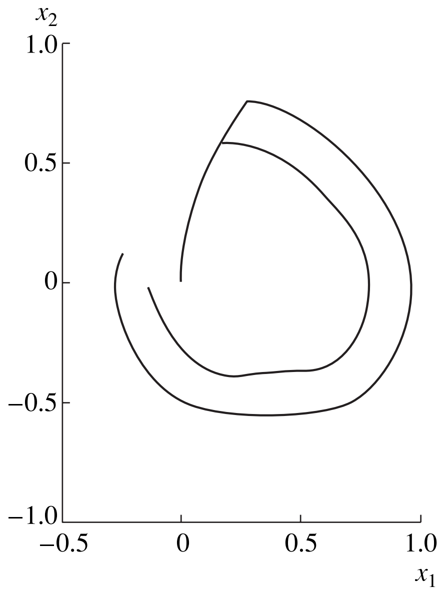

Fig. 1.

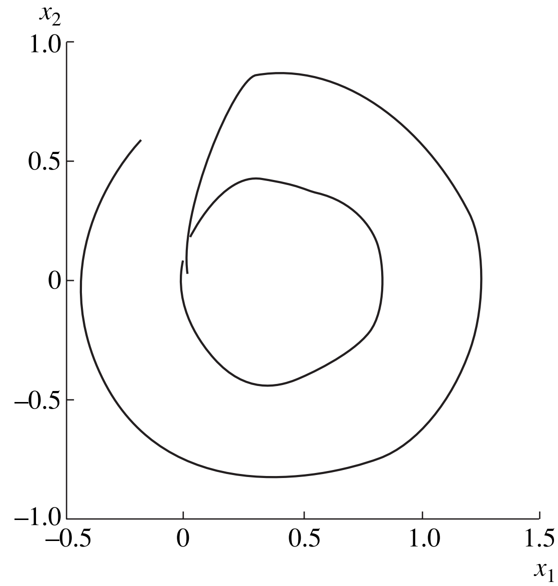

Fig. 2.

$$x_1(0) = x_2(0) = 0, \quad x_1(2\pi) \geq -3, \tag{3.1}$$

$$x_2(2\pi) \geq -0.3, \quad 0 \leq u(t) \leq 1, \quad t \in T = [0, 2\pi).$$

Introducing the new variable $\dot{x}_3 = u$, $x_3(0) = 0$, we can write problem (3.1) in the form (1.1)–(1.4).

The following two types of controls were constructed for this problem: (I) the guaranteeing optimal control without use of trial signals [14]; (II) the guaranteeing optimal control with the use of trial signals.

The control of type I was constructed on the basis of the information about the fact that the range of possible values of the disturbance is defined by the inequality $|w(t)| \leq 0.5$, $t \in T$. The constructed open-loop control has the form

$$u^0(t) = \begin{cases} 1, & t \in [0, 1.0159) \cup [5.2672, 2\pi), \\ 0, & t \in [1.0159, 5.2672). \end{cases}$$

The guaranteed value of the performance index on this control is $J_0^0 = 2.03$.

The defining equations of the optimal controller with the use of trial signals on the first interval of constancy of the structure have the following forms:

$$\sin t_1(\tau) - \sin t_2(\tau) - \sin\tau + 0.3 + \cos\tau x^*(\tau)$$

$$-0.1|\sin\tau| + \int_\tau^{2\pi} \cos t \tilde{w}_\tau^*(t)dt = 0,$$

$$y_2(\tau)\cos t_1(\tau) = -1,$$

$$y_2(\tau)\cos t_2(\tau) = -1.$$

The unknowns are $t_1(\tau)$, $t_2(\tau)$, and $y_2(\tau)$. An instant $\bar{\tau}$ comes in the course of the process when $t_1(\bar{\tau}) = \bar{\tau}$. The structure of the defining equations changes at this

Table

No.		1	2	3
Disturbance		$0.4\sin 0.5t + 0.1\sin 4t$	$0.4 + 0.1\sin 4t$	$0.2 + 0.2\sin 2t + 0.1\sin 4t$
Trial signal		$0.4\sin 0.5t$	0.4	$0.2 + 0.2\sin 2t$
Error $\tilde{v}_\tau(\cdot)$		0.1	0.1	0.1
Control I: J^*		0.72	0.62	0.65
Control II	t_i	0.623, 5.661	0.05, 6.233	0.322, 5.9609
	J_0^0	1.246	0.1	0.644
	J^*	0.54	0.06	0.3

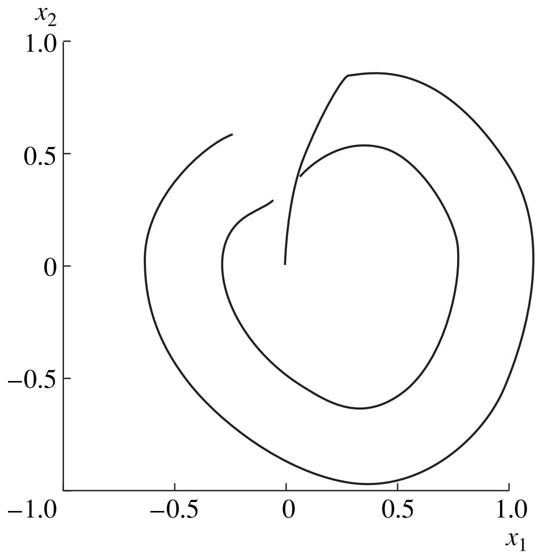

Fig. 3.

instant, and, for $\tau > \bar{\tau}$, the defining equations have the form

$$-\sin t_1(\tau) + 0.3 + \cos\tau x^*(\tau) - 0.1|\sin\tau| + \int_\tau^{2\pi} \cos t \tilde{w}_\tau^*(t)dt = 0,$$

$$y_2(\tau)\cos t_1(\tau) = -1.$$

The results of the operation of controllers of types I and II, under the action of various disturbances on the system, are presented in the table. The following notation is adopted there: t_i are the switching points of the optimal guaranteeing control of type II, J_0^0 is the guaranteed value of the performance index, J^* is the value of the performance index that was reached as a result of operation of controllers. In all the cases, the control produced by the controllers has the form

$$u^*(t) = \begin{cases} 1, & t \in [0, t^0), \\ 0, & t \in [t^0, 2\pi). \end{cases}$$

The value t^0 coincides with the value J^* of the performance index in the process under consideration.

The phase trajectories of systems that are controlled without use of trial signals (Curves I) and those controlled with the use of trial signals (Curves II) are presented in Figs. 1–3.

Remarks to the example In the example under consideration, the parameters of trial signals are assumed to be exclusively constant in order to simplify calculations. In actual practice, these parameters, as a rule, will be continuously varying with time, depending on the incoming information. However, such a generalization does not fundamentally affect the operation of the optimal controller. The law of variation of parameters will be different for distinct methods of date processing, and, in order not to give preference to any of them, we took the trial signal with constant parameters in this paper. Of course, the results of operation of the optimal controller based on actual signals will be somewhat poorer, but this cannot refute the principal conclusion on the advantages of the feedback with the use of trial signals over the feedback that does not use them.

CONCLUSION

The problem of implementation of the optimal feedback control in a modern statement is studied in this paper. This statement differs from the conventional one since the former is based on the mathematical model with disturbances. Of two possible types of disturbances, we consider the nonstochastic ones, which makes natural the problem of guaranteed optimization. The guaranteed optimal control problems in a modern statement admit several types of optimal feedbacks. The paper originally studies the case where it is possible to make a prediction concerning the future realizations of disturbances in the process of control. It is shown how a new type of the optimal feedback can be implemented in the real-time mode using modern computers. The illustrative examples presented show that the use of the additional information improves the result that was obtained by using the optimal feedback constructed by the method without use of the prediction proposed by the authors earlier.

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